Solar-System Experiments and Saa's Model of Gravity with Propagating Torsion

Fiziev P.[*], Yazadjiev S.[†]

Department of Theoretical Physics, Faculty of Physics,
Sofia University,
5 James Bourchier Boulevard, Sofia 1164,
Bulgaria

February 7, 2008

Abstract

It's shown that Saa's model of gravity with propagating torsion is inconsistent with basic solar-system gravitational experiments.

Recently a new model of gravity involving propagating torsion was proposed by A. Saa [1]-[5]. In the present note we investigate the consistency of this model with the basic solar-system gravitational experiments.

The main idea of the model is to replace the usual volume element $\sqrt{-g}d^4x$ with a new one – $e^{-3\Theta}\sqrt{-g}d^4x$, which is covariantly constant with respect to the transposed affine connection ∇^T. By virtue of the using a new volume element the equations of the motion for the matter and gauge fields are of an autoparallel type [1]-[5], [6].

In presence of spinless matter only in this model an Einstein-Cartan geometry with semi-symmetric torsion tensor $S_{\alpha\beta}{}^{\gamma} = S_{[\alpha}\delta^{\gamma}_{\beta]}$ is considered, where the torsion vector S_α is potential, i.e. there exists a potential Θ such that $S_\alpha = \partial_\alpha\Theta$. The following new equations for geometrical fields (metric and torsion) are obtained [1]-[5], [6]:

$$G_{\mu\nu} + \nabla_\mu\nabla_\nu\Theta - g_{\mu\nu}\Box\Theta = \tfrac{\kappa}{c^2}T_{\mu\nu},$$
$$\nabla_\sigma S^\sigma = \Box\Theta = -\tfrac{2\kappa}{c^2}\left(\mathcal{L}_M - \tfrac{1}{3}\frac{\delta\mathcal{L}_M}{\delta\Theta} + \tfrac{1}{2}T\right). \qquad (1)$$

[*]E-mail: fiziev@phys.uni-sofia.bg
[†]E-mail: yazad@phys.uni-sofia.bg

Here $G_{\mu\nu}$ is the Einstein tensor for the affine connection ∇, $\mathcal{L}_M$ is the matter lagrangian density, T is the trace of the energy-momentum tensor $T_{\mu\nu}$, and κ is the Einstein gravitational constant.

It is not hard to see that the equations (1) for geometric fields $g_{\alpha\beta}$ and Θ in vacuum coincide with the corresponding equations in Brans-Dicke theory [7], [8] in vacuum with parameter $\omega = -\frac{4}{3}$ if the field Θ in the Saa's model is replaced with a Brans-Dicke scalar field $\Phi = e^{-3\Theta}$. Hence, the asymptotically flat, static and spherically symmetric general solution of the vacuum geometric fields equations (1) is known [8], [9]. In isotropic coordinates it's given by a two-parameter – (r_0, k) family of solutions

$$ds^2 = \left(\frac{1-\frac{r_0}{r}}{1+\frac{r_0}{r}}\right)^{\frac{2}{\rho(k)}} (c\,dt)^2 - \left(1-\frac{r_0^2}{r^2}\right)^2 \left(\frac{1-\frac{r_0}{r}}{1+\frac{r_0}{r}}\right)^{\frac{2}{\rho(k)}(3k-1)} \left(dr^2 + r^2 d\Omega^2\right), \quad (2)$$

where $\rho(k) = \sqrt{3\left(k-\frac{1}{2}\right)^2 + \frac{1}{4}}$.

The parameter k presents the ratio of the torsion force (as defined in [6]) and the gravitational one [10]. In the case when $k = 0$ we have the usual torsionless Schwarzschild's solution and $r_0 \equiv 4r_g$ is the standard gravitational radius.

The week field approximation used in [7] yields the formal value $k = \frac{1}{2}$ when the source of the geometrical fields (metric and torsion) is a point with mass M, but this approximation is not valid in general for the case of a star [7]. Moreover, in Saa's model no result like Birkhoff theorem in general relativity take place. Therefore it is impossible to use all results for fields in vacuum obtained in the case of a point source for the fields in vacuum when the source of these fields is a massive star. Hence, we are not able to reject immediately Saa's model using the well known facts about the relation of the Branse-Dicke theory with solar-system experiments. A further study of the problem is needed.

The asymptotic expansion of the metric (2) at $r \to \infty$ is

$$ds^2 \approx \left(1 - \frac{4r_0}{\rho(k)r} + \frac{8r_0^2}{\rho(k)^2 r^2}\right)(c\,dt)^2 - \left(1 + (1-3k)\frac{4r_0}{\rho(k)r}\right)\left(dr^2 + r^2 d\Omega^2\right). \quad (3)$$

In the model under consideration, as it was shown in [6], the test particles move along geodesic lines. Therefore the active gravitational mass "seen" by the test particles is

$$M = \frac{2r_0}{\rho(k)}$$

and correspondingly the asymptotic expansion (3) may be written in the form

$$ds^2 \approx \left(1 - \frac{2M}{r} + 2\frac{M^2}{r^2}\right)(c\,dt)^2 - \left(1 + 2(1-3k)\frac{M}{r}\right)\left(dr^2 + r^2 d\Omega^2\right). \quad (4)$$

From here, it immediately follows that the first two of the post-Newtonian parameters are

$$\beta = 1, \gamma = 1 - 3k.$$

The solar-system experiments set tight constrains on the post-Newtonian parameters [11]:

$$\mid \beta - 1 \mid < 1 * 10^{-3}, \mid \gamma - 1 \mid < 2 * 10^{-3}.$$

Therefore Saa's model will not contradict to the experimental facts if the parameter k satisfies the inequality:

$$\mid k \mid < \frac{2}{3} * 10^{-3}.$$

In order to specify the theoretically possible values of k in Saa's model we must consider a model of a star with mass M as a source of geometrical fields[1]. Such a model was investigated in [10]. It was shown that the parameter k satisfies the constraint

$$\frac{1}{3} \leq k \leq \frac{1}{2}$$

under general physical assumptions: 1) spherically symmetric and stationary star state; 2) regularity of the solution at the center of the star; and 3) validity of the positivity condition $T = \varepsilon - 3p \geq 0$, ε being the energy density of the star, p being the pressure. The last condition is critical and means that the star is build of a normal matter. The parameter k takes maximal value $\frac{1}{2}$ (which coincide with its value obtained in the week field limit) in the case of nonrelativistic matter $\varepsilon \gg p$ and minimal value $\frac{1}{3}$ in the ultrarelativistic case $\varepsilon = 3p$.

Consequently Saa's model is inconsistent with basic solar-system gravitational experiments[2].

Conclusions

Saa's model seems to be interesting because in the metric-affine-connected spaces with nonzero torsion and curvature it leads to a consistent application of the minimal coupling principle both in the action principle and in the equations of motion for fields (the last being of an autoparallel type) [1]-[5]. This property of the model is not valid in the same spaces when one applies the minimal coupling principle both in the action principle and in the equations of motion (of a geodesic type) for particles [6]. Most probably the last inconsistency is responsible for the negative result we obtained in the present note. Thus we see that to comply Saa's model with the experimental data, if this is possible at all, we must try to change properly this model.

[1] In presence of matter the field equations of this model differ essentially from the corresponding equations in Branse-Dicke theory. Hence, an independent consideration of a star in Saa's model is needed

[2] For a matter with unusual properties when $\varepsilon \approx p$ (i.e. $T = \varepsilon - 3p < 0$ [12], [13]) we have $k \approx 0$ and Saa's model may not contradict to observations, but this is not the case of the solar system.

Acknowledgments

This work has been partially supported by the Sofia University Foundation for Scientific Researches, Contract No. 245/98, and by the Bulgarian National Foundation for Scientific Researches, Contract F610/98.

References

[1] Saa A 1997 Gen. Rel. and Grav. **29** 205

[2] Saa A 1993 Mod. Phys. Lett. **A8** 2565

[3] Saa A 1994 Mod. Phys. Lett. **A9** 971

[4] Saa A 1995 Class. Quant. Grav. **12** L85

[5] Saa A 1995 J. Geom. and Phys. **15** 102

[6] Fiziev P *Spinless matter in Transposed-Equi-Affine theory of gravity*, E-print gr-qc/9712004

[7] Brans C, Dicke R H 1961 Phys. Rev. **124** 925

[8] Brans C 1961 Phys. Rev. **125** 2194

[9] Xanthopoulos B C, Zannias T 1989 Phys. Rev. **D40** 2564

[10] Boyadjiev T, Fiziev P, Yazadjiev S *Neutron star in presence of torsion-dilaton field*, E-print gr-qc/9803084

[11] Will C M 1993 *Theory and Experiment in Gravitational Physics*, Cambridge University Press

[12] Zel'dovich Ya B 1961 Zh. Eksp. Teor. Fiz. **37** 569

[13] Zel'dovich Ya B, Novikov I D 1971 *Stars and Relativity*, The University of Chicago Press